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Pressure Waves in Volume Effect in Gas-Turbine Transient-Performance Models

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Gas-turbine volumes behave like air reservoirs and have an effect on gas-turbine transient behavior. The intercomponent volume (ICV) method has been widely used in gas-turbine transient performance research. It takes into account the volume effect and gives reasonable results in transient performance predictions. The introduction of a pressure waves in volume (PWV) effect based on the physics that the pressure propagation in a volume is the result of pressure wave movement is described. This effect is introduced in the ICV method for gas-turbine transient predictions. This method is applied to a two-spool turbojet engine. The detailed process of propagation of aerodynamic parameters in the engine rear duct during an acceleration is described. Compared with the original ICV method, the transient performance of the engine with the PWV method shows a small but interesting difference.

Nomenclature

A	=	area
a	=	speed of sound
FN	=	net thrust
M	=	Mach number
$\dot{m}$	=	mass flow rate
$\dot{m}_f$	=	fuel flow rate
N	=	rotational speed
n	=	polytropic index
P	=	pressure
R	=	gas constant
s	=	entropy
T	=	temperature
$T4$	=	turbine entry temperature
u	=	velocity
W	=	turbine work
γ	=	ratio of specific heats of gas
η	=	thermal efficiency
ρ	=	gas density

Subscripts

A, B, C, D	=	either side of a pressure wave
amb	=	ambient
max	=	maximum
ref	=	reference value
T	=	turbine
t	=	total
0	=	undisturbed area in front of a pressure wave
1	=	with intercomponent volume method
2	=	with pressure waves in volume method
5	=	low-pressure (LP) turbine inlet
6	=	LP turbine exit

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Introduction

IN the prediction of transient behavior of gas turbine engines, two methods are commonly used: the continuity of mass flow (CMF) method and the intercomponent volume (ICV) method, which were described in detail by Fawke and Saravanamuttoo.[1] The former does not take into account the volume effect, and it is assumed that at any given instant the mass flow into a component is matched with the mass flow emerging from it.

In the ICV method, it is assumed that volumes exist between adjacent components that allow the accumulation or release of air or gas. These volumes are representative of the volume of each component. The ICV method is a more accurate description of transient processes because this procedure includes the local mass accumulation or release in engine components, which is ignored in the CMF method. In practice, the CMF method is applicable to cases where large speed transients take place. The ICV method is more general, but it has a larger computational cost.

This paper introduces a pressure-wave volume (PWV) method to describe the pressure propagation inside a component volume. A comparison between the original ICV method and the PWV method is made with a prediction of the acceleration of a two-spool turbojet engine.

ICV and PWV Methods

The current state of the art in the ICV method[1] is that the mass flow into a component will not be the same as that out of the component in any transient instant. Engine components have associated volumes, thus air/gas mass will accumulate or diminish and the pressure in that volume will rise or fall at that instant. At the same time, it is assumed that the pressure propagates throughout the volume immediately, so that the pressure at any place inside the volume is the same at each time interval. This prompt pressure propagation assumption is not accurate, and it can alter the transient behavior of the engine, in particular in large volumes.

For example, in the volume between the low-pressure (LP) turbine and the nozzle of a gas turbine, pressure waves are generated from the turbine when the working conditions of the gas turbine change. The waves move downstream toward the nozzle, are reflected from the nozzle, and move upstream. When meeting the turbine, the waves are reflected and move downstream again. During the movement, two approaching pressure waves will pass through one another and become two new waves moving in opposite directions. All of the pressure waves degrade after they are reflected several times in the duct volume and eventually disappear. As a result of the wave movement, any changes of the aerodynamic parameters, such as pressure, temperature, and mass flow rate, from both ends of the duct volume propagate throughout the volume. This physical process is the theoretical basis of the PWV method.

The pressure wave movement delays the pressure propagation inside the volume, and the predicted gas turbine performance response with the PWV method during transient processes will be different when compared to the predictions made using the original ICV method.

Description of PWV Method

The largest volume in a turbojet engine is normally the rear duct behind the turbine. In the present analysis, any components inside the engine duct are ignored. The modeling of pressure wave generation, propagation, passing through other pressure waves, and reflection from both ends of the duct is described hereafter based on fluid dynamics.[2]

Two Sides of a Pressure Wave

When a pressure wave moves inside a duct volume in an axial direction, it is assumed that the gas passes through a pressure wave isentropically and that the wave propagates at the speed of sound. Equation (1) can be obtained with the momentum continuity, applying to a control volume surrounding a pressure wave in a relative frame of reference (Fig. 1):

$$\mathrm{d}u = \frac{1}{a}\frac{\mathrm{d}p}{\rho} \tag{1}$$

The relationship between the local sonic velocity and the property change rate is

$$a^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho} \tag{2}$$

Combining Eqs. (1) and (2) gives

$$\mathrm{d}u = a\frac{\mathrm{d}p}{\gamma P} \tag{3}$$

Then

$$a/a_0 = (T/T_0)^{\frac{1}{2}} = (P/P_0)^{(\gamma-1)/2\gamma} \tag{4}$$

Integrating Eq. (3) and substituting from Eq. (4) gives

$$u = u_0 + [2a_0/(\gamma-1)]\left[(P/P_0)^{(\gamma-1)/2\gamma} - 1\right] \tag{5}$$

where subscript 0 means undisturbed gas in front of the wave.

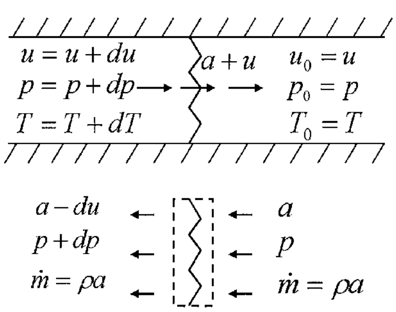

Fig. 1 Control volume: pressure wave in a duct.

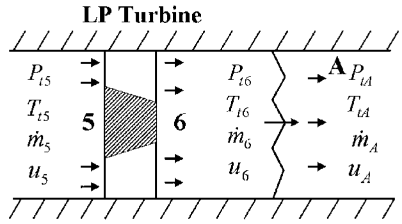

Fig. 2 Generation of a wave.

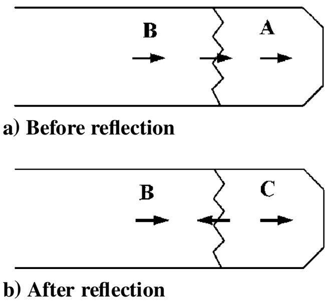

Fig. 3 Wave reflection from nozzle.

Pressure Wave from the Turbine

During transients, the LP turbine's working point keeps changing and the gas conditions, such as gas temperature, pressure, and mass flow rate, change accordingly. In a numerical process, it is assumed that a pressure wave is generated from the LP turbine exit in every numerical time step (Fig. 2).

With the turbine inlet parameters, the exit total pressure P_{t6} and the efficiency η_T the turbine work W_T can be found. With the turbine characteristics with $N/\sqrt{(T_{t5})}$ and W_T, a new turbine efficiency η_T and mass flow rate $\dot{m}_T\sqrt{(T_{t5})}/P_{t5}$ can be obtained. If the calculated mass flow rate is not the same as the inlet value, the turbine exit total pressure p_{t6} needs to be adjusted until both values are in agreement.

It is also assumed that the expansion process in the turbine is a polytropic process. With the application of the polytropic relation (6), the mass flow expression (7), and wave Eq. (8) into the newly generated pressure wave at the turbine exit, combined with relations between total and static parameters, turbine exit parameters T_{t6}, $\dot{m}_6$, and u_6 can be obtained:

$$\frac{T_{t6}}{T_{t5}} = \left(\frac{P_{t6}}{P_{t5}}\right)^{(n-1)/n}, \qquad \frac{n-1}{n} = \eta_T\frac{\gamma-1}{\gamma} \tag{6}$$

$$\dot{m}_6 = \frac{P_6 A_6 u_6}{R T_6} \tag{7}$$

$$u_6 = u_A + \frac{2a_A}{\gamma-1}\left[\left(\frac{P_6}{P_A}\right)^{(\gamma-1)/2\gamma} - 1\right] \tag{8}$$

In practice, rater than Eq. (6), the appropriate enthalpy–entropy relations would be used. The parameters on both sides of the pressure wave are those at the exit of the turbine in two successive time intervals.

Pressure Wave Reflection from the Nozzle

The reflection of a pressure wave from a nozzle is illustrated in Fig. 3, where Fig. 3a shows a pressure wave approaching the nozzle and Fig. 3b the wave leaving the nozzle after reflection.

The inlet conditions (P_t and T_t), the nozzle area, and ambient conditions (P_{amb} and T_{amb}) determine the duct flow. The nozzle may be unchoked or choked. It is assumed that the parameters in zone A and B are known and the reflection process from A to C is isentropic. The parameters in zone C, in the unchoked case, may be expressed with

$$P_c = P_{\mathrm{amb}}\left[\left(\frac{A_n}{A}\right)^2 \frac{(P_t/P_{\mathrm{amb}})^{(\gamma-1)/\gamma}-1}{(P_t/P_c)^{(\gamma-1)/\gamma}-1}\right]^{\gamma/(\gamma+1)} \tag{9}$$

$$T_C = T_A\left(\frac{P_C}{P_A}\right)^{(\gamma-1)/\gamma} \tag{10}$$

$$u_C = u_B - \frac{2a_B}{(\gamma-1)}\left[\left(\frac{P_C}{P_B}\right)^{(\gamma-1)/2\gamma} - 1\right] \tag{11}$$

$$\dot{m}_C = \sqrt{\frac{\gamma}{R}}\frac{P_C A}{\sqrt{T_C}}M_C \tag{12}$$

If a pressure wave is reflected from a choked nozzle, it is supposed that the effective mass flow rate remains unchanged and that Eq. (13) is satisfied:

$$\frac{\dot{m}_A\sqrt{T_{tA}}}{P_{tA}} = \frac{\dot{m}_C\sqrt{T_{tC}}}{P_{tC}} \tag{13}$$

Therefore,

$$M_A = M_C \tag{14}$$

and other parameters in zone C can be obtained with Eqs. (15–17)

$$P_C = \left\{\frac{u_B + [2a_B/(\gamma-1)]}{u_A(T_B/T_A)^{\frac{1}{2}} + 2a_B/(\gamma-1)}\right\}^{2\gamma/(\gamma-1)} \cdot P_B \tag{15}$$

$$T_C = T_B\left(\frac{P_C}{P_B}\right)^{(\gamma-1)/\gamma} \tag{16}$$

$$\dot{m}_C = \frac{P_{tC}A_N}{\sqrt{T_{tC}}}\sqrt{\frac{\gamma}{R}}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} \tag{17}$$

Pressure Wave Reflection from a Turbine

When a pressure wave moves upstream and approaches the turbine and is reflected (Fig. 4), the process from A to C is assumed to be polytropic and the mass flow rate remains constant. It is also assumed that the parameters in zone A and B are known and that the state of turbine inlet remains unchanged during the reflection process.

The polytropic process of the flow in the turbine is shown in the T–s chart in Fig. 5. Parameters in zone C can be obtained with a set of Eqs. (18–20):

$$(T_C/T_A) = (P_C/P_A)^{(n-1)/n}, \qquad (n-1)/n = \eta_t[(\gamma-1)/\gamma] \tag{18}$$

$$\dot{m}_A = P_A u_A A/RT_A = P_C u_C A/RT_C = \dot{m}_C \tag{19}$$

$$u_C - u_B = [2a_B/(\gamma-1)][(P_C/P_B)^{(\gamma-1)/2\gamma} - 1] \tag{20}$$

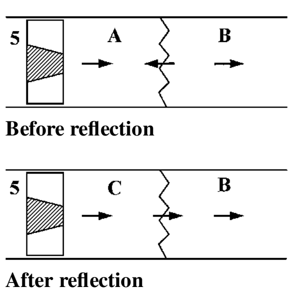

Fig. 4 Wave reflection from turbine.

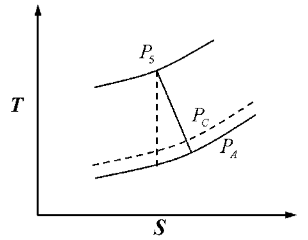

Fig. 5 Wave reflection from A to C.

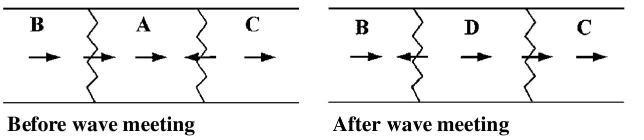

Fig. 6 Two waves passing through each other.

Pressure Waves Passing Through One Another

When two pressure waves in a duct approach each other and meet, each wave will go through the other and they will become two new waves moving in opposite directions. This process is assumed to be isentropic and the waves before and after they meet are illustrated in Fig. 6. Applying Eq. (5) to the pressure waves and combined with isentropic relations, the flow parameters in zone D can be calculated with Eqs. (21–24):

$$u_D = u_B - u_C + u_A \tag{21}$$

$$P_D = P_A\left\{[(\gamma-1)/2a_A](-u_D + u_B) + (P_B/P_A)^{(\gamma-1)/2\gamma}\right\}^{2\gamma/(\gamma-1)} \tag{22}$$

$$T_D = T_B(P_D/P_B)^{(\gamma-1)/\gamma} \tag{23}$$

$$\dot{m}_D = \left(P_D A M_D\big/\sqrt{T_D}\right)\sqrt{\gamma/R} \tag{24}$$

Dissipation of a Pressure Wave

After a pressure wave is reflected in a duct several times, it becomes weaker. A criterion has been set to define the dissipation of a pressure wave. It is assumed that when the pressure difference between both sides of a wave is small enough the wave has disappeared, and a set of average values of parameters is used to replace earlier parameters on both side of the wave. This criterion is defined in Eq. (25):

$$\Delta P < 0.001(kP_a) \tag{25}$$

Application to a Turbojet Engine

A transient performance prediction code has been modified to calculate engine transient performance with the ICV and PWV methods. Details of the transient prediction methods were described by Maccallum and Qi[3] and Pilidis.[4] The original code was used to predict the transient processes of Rolls-Royce Spey and Tay engines and provided satisfactory results.[5] Because of the unavailability of real engine transient performance data, a model two-spool turbojet engine has been used as an example to investigate the difference of the original ICV method and the PWV method. Both the PWV and the ICV methods were applied to the rear duct behind the turbine where the largest volume is located in the engine.

Two computational cases were carried out and compared with each other, one with the PWV method and another with the original ICV method. An acceleration process from idle to maximum fuel flow rate with heat transfer is selected for the analysis. In the two cases, the same fuel schedule was employed, where the nondimensional fuel flow is a function of high pressure (HP) compressor pressure ratio [Eq. (26)]. The fuel flow rate during an acceleration is shown in Fig. 7 where the fuel flow is increased to its maximum

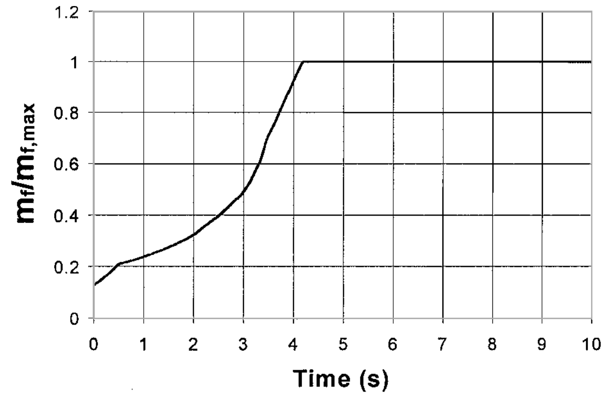

Fig. 7 Fuel flow during the transient.

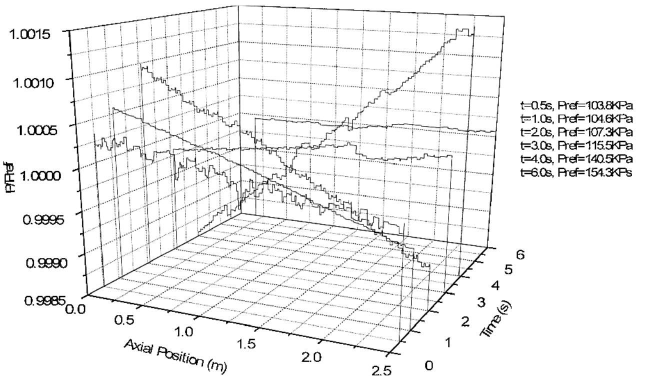

Fig. 8 Propagation history of pressure waves in the duct.

value in 4.2 s and then kept constant until the engine reaches the steady state:

$$\dot{m}_f / N_{\mathrm{HP}} P_{t2} = f(P_{t3}/P_{t2}) \tag{26}$$

It has been found that the transient performance difference predicted with the two volume methods is small. Hence, only the difference of the performance parameters during the transient are illustrated.

The pressure distributions in the duct at different time instants during the acceleration are illustrated in Fig. 8. In the first 2 s of the acceleration, a large number of compression waves are generated from the LP turbine and the propagation of the pressure change downstream of the turbine exit is delayed due to the movement of the pressure waves. The pressure upstream of the duct is higher than that downstream. After 3 s an increase of the downstream pressure can be observed, which is now higher than that upstream. This phenomenon lasts until about 2 s after the fuel flow rate reaches its maximum value. The engine condition will tend to the steady after the fuel flow rate reaches its maximum value and the pressure difference between the upstream and the downstream becomes progressively smaller. This means that the two volume methods give a similar result in that situation.

In the traditional ICV method, the pressure inside the duct volume has a value between the upstream and downstream pressure values predicted with the PWV method. The unsteady process of the pressure propagation inside the duct volume shown in Fig. 8 determines that the engine experiences an unsteady process, oscillating with low frequency around the process predicted with the original ICV method. This phenomenon is well illustrated with other parameters during the acceleration.

During the first 0.5 s of the acceleration, the LP turbine pressure ratio predicted with the PWV method (Fig. 9) is slightly larger (by about 0.15%) than that obtained with the ICV method. After that they are very similar. After about 2 s, the pressure ratio is lower than the value predicted with the ICV method by about 0.15%. After 4 s, the difference reaches a maximum (0.25%). This is maintained for

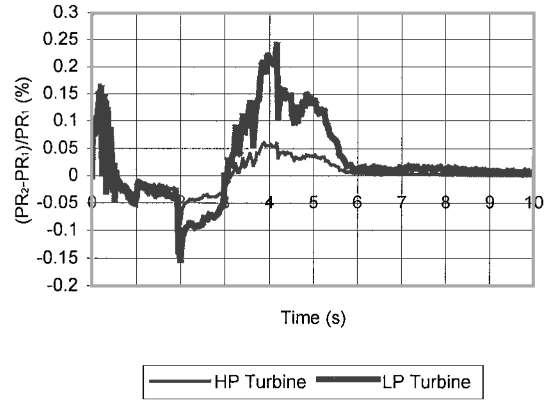

Fig. 9 Comparison of turbine pressure ratio.

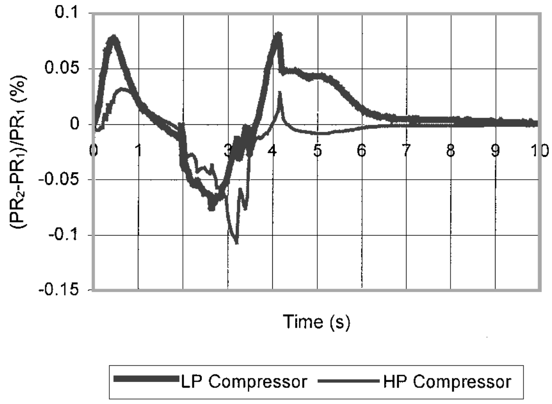

Fig. 10 Comparison of compressor pressure ratio.

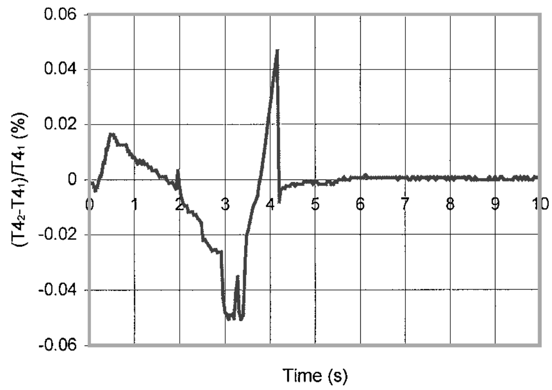

Fig. 11 $T4$ comparison.

about 2 s after the fuel flow reaches its maximum value. The high-pressure (HP) turbine pressure ratio shows a similar pattern, but with smaller magnitudes because the response is attenuated by the smaller volume of the LP turbine.

The compressor transient response is similar to that of the turbine (Fig. 10). The difference of HP turbine entry temperature between two volume methods during the transient is shown in Fig. 11. Although it shows a similar pattern to the pressure ratios, it is clear that the temperature difference is really small in magnitude, with a maximum difference of about 0.04%; therefore, the temperature difference can be neglected.

The difference of shaft speeds between the ICV and the PWV methods during the transient is shown in Fig. 12. In the first 1.5 s, the LP shaft speed predicted with the PWV method is higher than that predicted with the ICV method by about 0.11%. In the next 2 s it is lower by a maximum of about 0.075%. In the following 2.5 s is higher by a maximum of about 0.08%. The HP shaft speed predicted with the PWV shows somewhat smaller magnitudes with a maximum difference of about 0.04%.

The thrust difference between the two methods comes from the difference of the parameters at nozzle exit, the pressure, and mass flow rate. Figure 13 shows that in the first 1.5 s, the thrust predicted with the PWV method is delayed by about 2% in maximum compared with that predicted by the ICV method. After about 2 s the thrust with the PWV method is delayed again by about 0.08% in maximum compared to the thrust by ICV method. It can be seen from the preceding comparison that all of the performance parameters of the engine with the PWV method oscillate with low frequency around the transient performance predicted with the ICV method because of the oscillation of the pressure distribution inside the duct. As expected, volume size has an influence on the oscillation magnitude. For instance, in the same numerical test when the duct length is halved, the difference between the shaft speeds and thrust predicted with the two methods is reduced (Figs. 14 and 15).

The physical process of the transient flow in the duct volume in the engine is better described with the PWV method than the ICV method based on the theory of flow dynamics. Unfortunately, the accuracy of the predicted pressure wave effect on gas-turbine transient performance is difficult to validate due to a lack of testing programs and data.

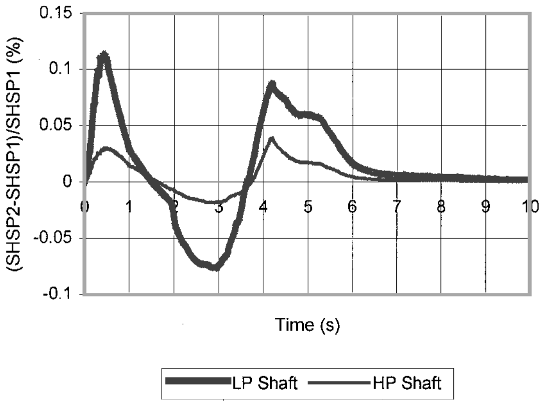

Fig. 12 Shaft speed comparison.

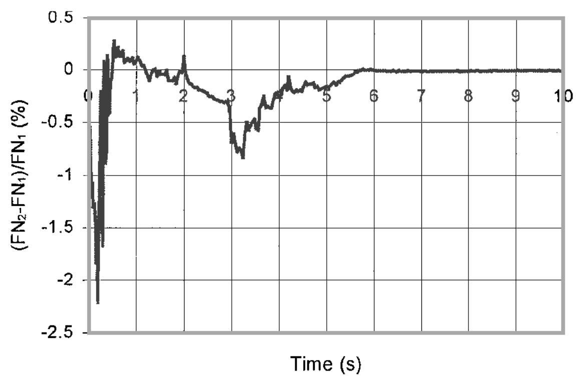

Fig. 13 Thrust comparison.

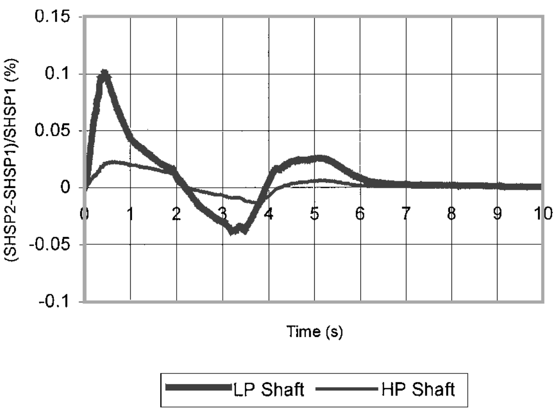

Fig. 14 Shaft speed comparison (duct length halved).

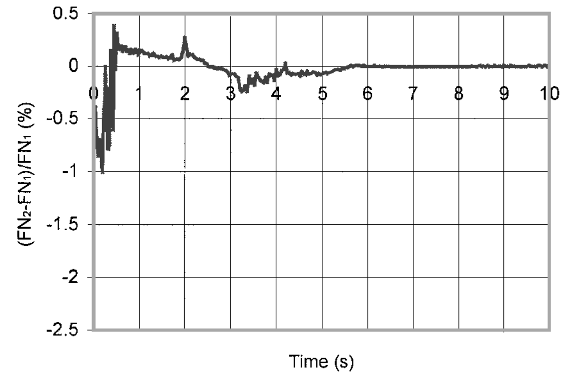

Fig. 15 Thrust comparison (duct length halved).

Pressure waves may show different transient effects on engines with different geometries, such as turbofan engines with large bypass ducts and industrial gas-turbines with long duct pipes. Lower temperature at high altitude may also contribute to larger pressure-wave effects on the transient performance of aeroengines. These effects will be studied further in the future.

An alternative analysis for the high-frequency effects just described has been resented by Merriman.[6] The conclusions reported by the present authors are in general agreement with those of Merriman.

More computational effort is required with the PWV method than with the ICV method. For this specific transient process calculation with a DEC ALPHA FARM computer, about 5 s in CPU is needed with the ICV and about 8 min with the PWV. The computation time is very sensitive to the length of the duct.

Conclusion

The PWV method described here is a better description of gas-turbine transient performance than the original ICV method because it includes the effect of pressure propagation inside the volumes. The predicted transient response of a turbojet engine with the PWV method is slightly different from that with the original ICV method.

The difference predicted here is small, but it may be significant when large volumes such as bypass ducts are considered. Similarly, some large industrial gas turbines have very large volumes too.

No experimental verification was carried out; however, the authors believe that the inclusion of this effect must be actively pursued because on many occasions the effects could be significant in some critical maneuvers such as altitude relight and reheat lighting in a low-bypass turbofan.

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